Objective To introduce and provide practice with a “low-stress” division algorithm for 1-digit divisors.

**1 Teaching the Lesson**

**Key Activities**

Students learn and practice a paper-and-pencil algorithm for division that permits them to build up the quotient by working with “easy” numbers.

**Key Concepts and Skills**

- Identify and use multiples of 10. (Number and Numeration Goal 3)
- Add multiples of 10. (Operations and Computation Goal 1)
- Subtract multidigit numbers. (Operations and Computation Goal 2)
- Apply extended multiplication facts to long-division situations. (Operations and Computation Goal 3)
- Solve equal-grouping division number stories. (Operations and Computation Goal 4)

**Key Vocabulary**

- dividend
- divisor
- partial quotient

**Ongoing Assessment:**

- **Informing Instruction** See pages 413 and 416.
- **Recognizing Student Achievement** Use journal pages 144 and 145. (Operations and Computation Goal 4)

**2 Ongoing Learning & Practice**

Students solve problems involving various basic skills with decimals.

Students practice and maintain skills through Math Boxes and Study Link activities.

**materials**

- Math Journal 1, pp. 144 and 145
- Study Link 6-2
- Teaching Aid Masters (Math Masters, pp. 403, 436, and 438) (optional)
- slate

See Advance Preparation

**3 Differentiation Options**

**READINESS**

Students play Beat the Calculator with extended multiplication facts.

**ENRICHMENT**

Students use clues to solve a division number story.

**EXTRA PRACTICE**

Students play Division Dash to practice dividing 2- or 3-digit dividends by 1-digit divisors.

**ELL SUPPORT**

Students display and label parts of division number models.

**materials**

- Math Journal 1, pp. 146 and 147
- Study Link Master (Math Masters, p. 180)
- Student Reference Book, pp. 233 and 241
- Teaching Master (Math Masters, p. 181)
- Game Masters (Math Masters, pp. 461 and 471)
  - per partnership: 4 each of number cards 1–9
  - per group: 4 each of number cards 1–10
  - chart paper; colored markers
  - 10 nickels (optional)

**Additional Information**

**Advance Preparation** Make 1 or 2 copies of Math Masters, page 403 for each student. Have extra copies available throughout this unit.

**Technology**

- Assessment Management System
  - Journal pages 144 and 145, Problems 1, 2, and 5
  - See the iTLG.

412 Unit 6 Division; Map Reference Frames; Measures of Angles
Getting Started

Mental Math and Reflexes
Pose true or false problems involving equal groups. Suggestions:

- True: There are at least 2 [5s] in 11.
- False: There are at least 3 [2s] in 5.
- True: There are at least 4 [3s] in 14.

Study Link 6-2 Follow-Up
Partners compare answers. Have volunteers explain different strategies that could be used to solve Problem 3.

1 Teaching the Lesson

Math Message Follow-Up

In the first problem, the number of weeks is known; students find the number of days by multiplying by 7. In the second problem, the total number of days is given; students divide by 7 to find the number of weeks.

<table>
<thead>
<tr>
<th>weeks</th>
<th>days per week</th>
<th>days in all</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>7</td>
<td>210</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>weeks</th>
<th>days per week</th>
<th>days in all</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>7</td>
<td>98</td>
</tr>
</tbody>
</table>

Introducing the Partial-Quotients Algorithm
(Math Masters, pp. 403 and 438)

This lesson formally introduces the partial-quotients algorithm. Begin with an equal-grouping division problem like the following:

- Amy is 127 days older than Bob. How many weeks older is Amy?

Briefly work through the steps mentioned in the previous lessons:

Step 1: Decide what you need to find out. How many weeks older is Amy?
Step 2: Identify the data you need to solve the problem. The number of days in all and the number of days per week. Write the data in the appropriate spaces in the diagram.

<table>
<thead>
<tr>
<th>weeks</th>
<th>days per week</th>
<th>days in all</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>7</td>
<td>127</td>
</tr>
</tbody>
</table>

Step 3: Decide what to do to find the answer. 127 days must be equally grouped (divided) so that there are 7 days in each week.

Step 4: Do the computation. Model the following algorithm while students follow along with paper and pencil.

1. Write the problem in the traditional form: \( \frac{7}{127} \). Point out that the **dividend**—the number that is being divided—is 127. The **divisor**—the number that the dividend is being divided by—is 7. To support English language learners, label the dividend and the divisor on the board.

2. Draw a vertical line so that the problem looks like the problem below. The vertical line will separate subtractions from partial quotients.

\[ 7 \longdiv{127} \]

3. Suggest that one way to proceed is to use a series of “at least/not more than” multiples of the divisor. A good strategy is to start with easy numbers, such as 100 times the divisor or 10 times the divisor.

   - Are there at least 100 \( [7s] \) in 127? No, because \( 100 \times 7 = 700 \), which is more than 127.
   - Are there at least 10 \( [7s] \) in 127? Yes, because \( 10 \times 7 = 70 \), which is less than 127.
   - Are there at least 20 \( [7s] \)? No, because \( 20 \times 7 = 140 \), which is more than 127.
   - So there are at least 10 \( [7s] \), but not more than 20 \( [7s] \). Try 10.

   Write 10 \( \times 7 \), or 70, under 127. Write 10 at the right. 10 is the first **partial quotient**. Partial quotients will be used to build up the final quotient.

\[
\begin{array}{c|c}
7 & 127 \\
7 & 127 \\
0 & 57 \\
\end{array}
\]

4. The next step is to find out how much is left to divide. Subtract 70 from 127.

\[
\begin{array}{c|c}
7 & 127 \\
0 & 57 \\
\end{array}
\]

**Adjusting the Activity**

Suggest that students first make a list of “easy” multiples of the divisor. For example, if the divisor is 7, students might make the following list:

- \( 100 \times 7 = 700 \)
- \( 50 \times 7 = 350 \)
- \( 20 \times 7 = 140 \)
- \( 10 \times 7 = 70 \)
- \( 5 \times 7 = 35 \)
- \( 2 \times 7 = 14 \)
- \( 1 \times 7 = 7 \)

Depending on the students and the divisor, the list can be extended or reduced. Students can use their list of easy multiples to take the guesswork out of successive multiples and focus on solving the division problem.

Students will realize that they can work more quickly if they begin with a more extensive list of multiples. *Math Masters*, page 438 provides an optional form for writing multiples.

**AUDITORY • KINESTHETIC • TACTILE • VISUAL**
5. Now find the number of 7s in 57. Following are two ways to do this:

▷ Use a fact family: \(8 \times 7 = 56\), so there are at least 8 [7s] in 57. Record as follows:

\[
\begin{array}{c|c}
7 & 127 \\
\hline
- & 70 \\
\hline
& 57 \\
\end{array}
\]

(The first partial quotient) \(10 \times 7 = 70\)

Subtract. 57 is left to divide.

\[
\begin{array}{c|c}
& 56 \\
\hline
& 8 \\
\end{array}
\]

(The second partial quotient) \(8 \times 7 = 56\)

▷ Use “at least / not more than” multiples with easy numbers. For example, ask:

● Are there at least 10 [7s] in 57? No, because \(10 \times 7 = 70\).

● Are there at least 5 [7s]? Yes, because \(5 \times 7 = 35\).

Next, subtract 35 from 57 and continue by asking:

● How many 7s are in 22? 3

\[
\begin{array}{c|c}
7 & 127 \\
\hline
- & 70 \\
\hline
& 57 \\
\end{array}
\]

(The first partial quotient) \(10 \times 7 = 70\)

Subtract. 57 is left to divide.

\[
\begin{array}{c|c}
& 35 \\
\hline
& 22 \\
\end{array}
\]

(The second partial quotient) \(5 \times 7 = 35\)

Subtract. 22 is left to divide.

\[
\begin{array}{c|c}
& 21 \\
\hline
& 3 \\
\end{array}
\]

(The third partial quotient) \(3 \times 7 = 21\)

6. For both ways, the division is complete when the subtraction leaves a number less than the divisor (7 in this example). The final step is to add the partial quotients—the numbers of 7s that were subtracted. 18 is the quotient. There is 1 left over. So, 1 is the remainder.

\[
\begin{array}{c|c}
7 & 127 \\
\hline
- & 70 \\
\hline
& 57 \\
\end{array}
\]

(The first partial quotient) \(10 \times 7 = 70\)

Subtract. 57 is left to divide.

\[
\begin{array}{c|c}
& 56 \\
\hline
& 8 \\
\end{array}
\]

Subtract. 57 is left to divide.

\[
\begin{array}{c|c}
& 18 \\
\hline
& 1 \\
\end{array}
\]

7. Have students record the final answer in the traditional position above the dividend. To support English language learners, label the quotient and the remainder.

\[
\frac{18}{7} \text{ R1}
\]

8. Conclude by interpreting the answer: Amy is 18 weeks and 1 day older than Bob.
Lead students through several more problems on the board. Ask: How many [ns] are there in m? Each n should be a 1-digit number; each m should be a 2- or 3-digit number. Some students may be ready for a 2-digit divisor. Suggestions:

- How many [4s] are there in 92? 23
- How many [3s] are there in 87? 29
- How many [7s] are there in 301? 43
- How many [8s] are there in 925? 115 R5
- How many [12s] are there in 588? 49
- How many [15s] are there in 556? 37 R1

Have the class pose a division problem, and ask partnerships to try to find the answer. Have volunteers share their work with the class. Look for students who got the same results in different ways.

**Ongoing Assessment: Informing Instruction**

Watch for students using multiples that are not too large and that are easy to work with. Using such multiples may require more steps, but it will make the work go faster. Also, students should not be concerned if they pick a multiple that is too large. If that happens, they will quickly realize that they have a subtraction problem involving a larger number below.

**Using the Partial-Quotients Algorithm**

*(Math Journal 1, pp. 144 and 145)*

Students use the partial-quotients algorithm to solve number stories and computation problems. Have copies of *Math Masters*, pages 436 and 438 readily available for students who choose to use them. Encourage students to use the relationship between multiplication and division to check their answers.

**Ongoing Assessment: Recognizing Student Achievement**

Use journal pages 144 and 145, Problems 1, 2, and 5 to assess students’ ability to solve division problems and number stories with 1-digit divisors and 2-digit dividends. Students are making adequate progress if they are able to compute the answers using the partial-quotients division algorithm. Some students may be able to solve Problems 3, 4, and 6–8, which involve 2-digit divisors or 3-digit dividends.
Ongoing Learning & Practice

Reviewing Place Value in Decimals
(Math Journal 1, p. 146)

Students solve problems involving ordering of decimals, naming place value in decimals, and reading and writing decimals.

Math Boxes 6-3
(Math Journal 1, p. 147)

Mixed Practice Math Boxes in this lesson are paired with Math Boxes in Lesson 6-1. The skill in Problem 5 previews Unit 7 content.

Writing/Reasoning Have students write a response to the following: Explain how the exponent in Problem 4a changes the value of 10. Sample answer: The exponent tells how many times the base 10 is used as a factor. For example, 10^4 = 10 * 10 * 10 * 10, or 10,000.

Study Link 6-3
(Math Masters, p. 180)

Home Connection Students practice using the partial-quotients division algorithm.
3 Differentiation Options

READINESS

Playing Beat the Calculator
(Student Reference Book, p. 233; Math Masters, p. 461)

To provide experience with multiplying extended facts, students play a version of Beat the Calculator in which the caller attaches a 0 to one or both of the factors shown on the cards.

Determination of the Cost of Pens
(Math Masters, p. 181)

To apply students’ understanding of division, have them use clues to find the cost of several pens. Encourage students to use money model the problem if necessary.

EXTRA PRACTICE

Playing Division Dash
(Student Reference Book, p. 241; Math Masters, p. 471)

To practice dividing 2- or 3-digit dividends by 1-digit divisors, have students play Division Dash. See Lesson 6-4 for additional information.

ELL SUPPORT

Building Vocabulary

To provide language support for division, have students write division number models on chart paper in the following ways:

\[
\begin{align*}
42 \div 5 & \rightarrow 8 \text{ R} 2 \\
42 & \div 5 \rightarrow 8 \text{ R} 2 \\
42 & \div 5 \rightarrow 8 \text{ R} 2
\end{align*}
\]

Have students do the following for each number model:

- Label and underline the dividend (number being divided) in red.
- Label and underline the divisor (the number the dividend is being divided by) in blue.
- Label and circle the quotient in a third color.
- Label and circle the remainder in a fourth color.

Point out that both the quotient and the remainder are part of the answer. Display this chart throughout the division lessons.
# The Partial-Quotients Division Algorithm, Part 2

**Objective** To provide practice with a “low-stress” division algorithm for 2-digit divisors.

## 1 Teaching the Lesson

### Key Activities
Students review and practice a paper-and-pencil algorithm for division that permits them to build up the quotient by working with “easy” numbers. Students focus on problems with 2-digit divisors.

### Key Concepts and Skills
- Identify and use multiples of 10. [Number and Numeration Goal 3]
- Add multiples of 10. [Operations and Computation Goal 1]
- Subtract multidigit numbers. [Operations and Computation Goal 2]
- Apply extended multiplication facts to long-division situations. [Operations and Computation Goal 3]
- Solve equal-grouping division number stories and problems. [Operations and Computation Goal 4]

#### Ongoing Assessment: Recognizing Student Achievement
Use journal page 166. [Operations and Computation Goal 4]

### materials
- Math Journal 1, pp. 166 and 167
- Study Link 6-9
- Teaching Aid Masters (Math Masters, p. 436, optional; p. 438)
- base-10 blocks
- slate

## 2 Ongoing Learning & Practice

Students take a 50-facts test. They use a line graph to record individual and class scores. Then students find the median and calculate the mean of all class scores.

Students practice and maintain skills through Math Boxes and Study Link activities.

### materials
- Math Journal 1, p. 168
- Study Link Master (Math Masters, p. 197)
- Teaching Aid Masters (Math Masters, pp. 413, 414, and 416)
- pen or colored pencil

## 3 Differentiation Options

### READINESS
Students play Division Dash to practice dividing 2- or 3-digit dividends by 1-digit divisors.

### ENRICHMENT
Students perform and explain a division “magic trick.”

### materials
- Student Reference Book, p. 241
- Teaching Master (Math Masters, p. 198)
- Game Master (Math Masters, p. 471)
- per partnership: 4 each of number cards 1–9

## Technology

Assessment Management System
Journal page 166, Problems 1 and 2
See the iTLG.

Lesson 6-10 455
Remind students that by packing 246 eggs into cartons, they are dividing 246 eggs into groups of 12. The problem is a division problem: How many [12s] are in 246?

Write the problem on the board in four of the ways that division problems can be written:

\[
\frac{246}{12} \quad \frac{246}{12} \quad \frac{246}{12} \quad \frac{246}{12}
\]

Ask several students to give their solutions and describe their strategies:

▷ Use a Multiplication/Division Diagram (Math Masters, page 436). Some students may think “What number times 12 equals 246?” while others may reason “246 divided by 12 equals what number?”

<table>
<thead>
<tr>
<th>cartons</th>
<th>eggs per carton</th>
<th>eggs in all</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>12</td>
<td>246</td>
</tr>
</tbody>
</table>

▷ Take 2 flats, 4 longs, and 6 cubes. Trade the flats for longs. Divide the longs and cubes into as many groups of 12 cubes as possible. Continue to exchange longs for cubes as necessary. 20 groups, 6 left over

▷ Draw a picture.
Break 246 into smaller “friendly numbers” such as the following:

- \(120 + 120 + 6 = 246\). There are 10 \([12s]\) in 120 and 10 \([12s]\) in 120. \(10 \div 10 = 10\), so there are 20 \([12s]\) in 246 with 6 left over.

- \(240 + 6 = 246\). There are 20 \([12s]\) in 246 with 6 left over.

You need 20 full cartons, plus one other carton to hold the 6 eggs that are left over. So, 21 cartons are needed.

Some students may have used the partial-quotients division algorithm to solve the Math Message problem. Ask a volunteer to explain how the algorithm was used.

### Introducing the Partial-Quotients Algorithm with 2-Digit Divisors

*Math Masters, p. 438*

Explain that most of the division problems in Unit 6 have involved a 1-digit divisor. In this lesson students will use the partial-quotients division algorithm to solve problems and number stories, like the Math Message, that involve 2-digit divisors.

The algorithm works the same whether you divide by a 2-digit or a 1-digit divisor. As it often helps to write down some easy facts for the divisor first, have copies of *Math Masters*, page 438 readily available. Remind students to use the relationship between multiplication and division to check their answers.

**Example 1:**

Teddy received a carton of 400 baseball cards from his grandmother. He decided to share them with his classmates. How many cards did Teddy and each of his 21 classmates get?

**One way:**

\[
\begin{array}{c|c|c|c}
22 & \text{400} & \text{22} & \text{400} & \text{22} & \text{400} \\
\hline
-220 & 10 & -220 & 10 & -220 & 10 \\
180 & 180 & 180 & 180 & \\
-110 & 5 & -110 & 5 & -176 & 8 \\
70 & 70 & 4 & 18 & \\
\hline
-22 & 1 & -66 & 3 & \\
48 & 4 & 18 & \\
-22 & 1 & \\
26 & 1 & \\
-22 & 1 & \\
4 & 18 & \\
\end{array}
\]

The answer, 18 R4, is the same for each method. Teddy and his classmates each received 18 baseball cards. There were 4 left over.
**Example 2:**

Sofia was playing with a set of 743 blocks. She decided to arrange them in stacks of 12. How many stacks did Sofia make?

**One way:**

<table>
<thead>
<tr>
<th>12743</th>
<th>12743</th>
<th>12743</th>
</tr>
</thead>
<tbody>
<tr>
<td>-600</td>
<td>-600</td>
<td>-600</td>
</tr>
<tr>
<td>143</td>
<td>143</td>
<td>143</td>
</tr>
<tr>
<td>-60</td>
<td>-120</td>
<td>-132</td>
</tr>
<tr>
<td>83</td>
<td>23</td>
<td>-1</td>
</tr>
<tr>
<td>-60</td>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>23</td>
<td>11</td>
<td>61</td>
</tr>
<tr>
<td>-12</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>11</td>
<td>61</td>
<td></td>
</tr>
</tbody>
</table>

The answer, 61 R11, is the same for each method. Sofia made 61 stacks of 12 blocks. There were 11 blocks left over.

**Using the Partial-Quotients Algorithm with 2-Digit Divisors**

(Math Journal 1, pp. 166 and 167; Math Masters, p. 438)

Students use the partial-quotients algorithm to solve division problems and number stories on journal pages 166 and 167.

**Ongoing Assessment:**

**Recognizing Student Achievement**

Use journal page 166, Problems 1 and 2 to assess students’ ability to solve problems involving the division of multidigit whole numbers by 1-digit divisors. Students are making adequate progress if they are able to calculate the quotients in Problems 1 and 2 and express the remainder in Problem 2 correctly. Some students may be able to solve Problems 3–9, which involve 2-digit divisors.

[Operations and Computation Goal 4]

**Taking a 50-Facts Test**

(Math Masters, pp. 413, 414, and 416)

See Lesson 3-4 for details regarding the administration of the 50-facts test and the recording and graphing of individual and class results.
Math Boxes 6-10

(Math Journal 1, p. 168)

Mixed Practice Math Boxes in this lesson are paired with Math Boxes in Lesson 6-8. The skill in Problem 6 previews Unit 7 content.

Writing/Reasoning Have students write a response to the following: True or false? (5,0) and (0,5) are both ordered number pairs that can be used to describe the location of point A in Problem 1. Explain your answer. False. The first number in an ordered number pair tells how far to go to the right, and the second number tells how far to go up. Only (0, 5) tells the location of point A.

Study Link 6-10

(Math Masters, p. 197)

Home Connection Students practice using the partial-quotients division algorithm for 2-digit divisors. For some problems, they may use a division method of their choice.

3 Differentiation Options

READINESS

Playing Division Dash

(Student Reference Book, p. 241, Math Masters, p. 471)

To provide experience with 2- or 3-digit dividends and 1-digit divisors in division problems, have students play Division Dash.

ENRICHMENT

Performing a “Magic Trick”

(Math Masters, p. 198)

To further explore the concept of division, have students use a calculator to perform a division “magic trick.”

Math Masters, page 198

Math Journal 1, p. 168

Math Boxes

1. Name the ordered number pair for each point plotted on the coordinate grid.
2. Plot the ordered number pair below.
3. Draw a line from point A to point B.
4. Write a number model for the following:
   - 180
   - 2,233
   - 1,827
   - 45 R6

Study Link Master

Division

1. Divide 14 gooses to make a small picture of jacks. Amelia has 100 gooses, how many pictures of jacks can she make?
   - Number model: 140 ÷ 10 = 14
   - Answer: 14 pictures of jacks

2. Each bouquet needs 21 flowers. The flower line has 400 flowers in the store, how many bouquets can the florist make?
   - Number model: 400 ÷ 21 = 19 R1
   - Answer: 19 bouquets

3. How many flowers are left over?
   - Flowers left over: 10

Math Masters, p. 197